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Mathematical modeling of the effect of screening for unaware HIV/AIDS-infected patients using homotopy perturbation method

Vijayakumar P.N.[1], Balaganesan P.[1], Rekha S.[1], Renuka J.[2]

[1]Academy of Maritime Education and Training (AMET) Deemed to be University, Chennai, Tamil Nadu, India
[2]Women's Christian College, Chennai, Tamilnadu, India

Abstract

Introduction: In this paper, we analyzed the study of a mathematical model of non-linear differential equation on the effect of HIV/AIDS disease among unaware HIV/AIDS-infected population.

Material and methods: Population was divided into four categories, including HIV-negative individuals, unaware HIV-positive cases, aware HIV-positive, and AIDS patients. The model was investigated numerically and analytically using fourth-order Runge-Kutta approach and homotopy perturbation method (HPM).

Results: We have discussed the parameter variation graphically.

Conclusions: Determining the dynamics of HIV prevalence and investigating the effect of each parameter on the governing equation can be simple with analytical solution.

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Key words: HIV/AIDS, AIDS patients, unaware infection, homotopy perturbation method.

Introduction

Throughout the world, people are infected with human immunodeficiency virus (HIV) that can lead to acquired immunodeficiency syndrome (AIDS). People who have had sexual contact with HIV-positive people, gay, and those who have been transfused with tainted blood, are the most likely to become infected. Mathematical models of HIV transmission dynamics are beneficial for higher knowledge of epidemiological patterns and sickness manipulation due to the fact they are able to assume HIV and AIDS occurrence within side quick and lengthy run.

In April, 1986, the first AIDS case was discovered in Cuba. This marked the beginning of the country's AIDS epidemic. At the end of 1985, some HIV seropositive cases were discovered. Earlier this year, the Cuban government began taking precautionary measures to prevent the onset of the disease. An universal ban on the import of blood and blood products was one of these measures [1]. To better understand the impact of a range of factors on the general pattern of AIDS epidemic, Anderson *et al.* [2] constructed a basic HIV transmission model. During the infection's chronic phase, in a homogeneously mixed community, Greenhalgh *et al.* [3] evaluated the influence of condom use on HIV/AIDS sexual transmission. The number of gay men in the United States is increasing. Rao [4] established a model for simulating the Indian AIDS epidemic. Naresh and Tripathi [5] looked at HIV infection transmission in

Address for correspondence: Balaganesan P., Academy of Maritime Education and Training (AMET) Deemed to be University, Chennai, Tamil Nadu, India, e-mail: balaganesanpp@gmail.com

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a tuberculosis-affected population. Piqueira *et al.* [6] created "A model of HIV/AIDS transmission in homosexual societies that integrates different approaches, blood screening, and social network influences".

The application of homotopy perturbation method in linear and non-linear issues has been adopted by researchers since this method continuously deform a simple problem. Homotopy perturbation method (HPM) was first proposed by He [7-9]. Moreover, Rajendran *et al.* [10] used unprecedented homotopy perturbation method for solving non-linear equations in the enzymatic reaction of glucose in a spherical matrix.

HIV/AIDS model description and formula

In this model [11], non-negative patients – $N(t)$, positive but unaware of being HIV-infected – $I_1(t)$, positive and aware of being HIV-infected – $I_2(t)$, and AIDS patients (the most advanced stage of HIV), were grouped into the four categories, which constituted HIV/AIDS population of this study, defined as $A(t)$. HIV/AIDS model was described by the system below of non-linear differential equations.

In the study of transmission dynamics, HIV/AIDS model was applied, and the corresponding parameter values are presented in Table 1, referred from [11].

$$\frac{dN}{dt} = Q_0 - dN - \alpha A \quad (1)$$

$$\frac{dI_1}{dt} = \frac{I_1\beta_1(N - I_1 - I_2 - A)}{N} + \frac{I_2\beta_2(N - I_1 - I_2 - A)}{N} - (\theta + \delta + d)I_1 \quad (2)$$

$$\frac{dI_2}{dt} = \theta I_1 - (\delta + d)I_2 \quad (3)$$

$$\frac{dA}{dt} = \delta I_1 + \delta I_2 - (\alpha + d)A \quad (4)$$

With the initial condition $N(0) \geq 0$, $I_1(0) \geq 0$, $A(0) \geq 0$. The model was well-posted for $N > 0$.

Analytical solution of HIV/AIDS transmission dynamics

In this section, a new approach homotopy perturbation method was used to solve analytical solution of equations (1)-(4), the expand derivative of HPM is discussed in Appendix A.

$$(1-Q)\left(\frac{dN}{dt} + dN\right) + Q\left(\frac{dN}{dt} - Q_0 + dN + \alpha A\right) = 0 \quad (5)$$

$$(1-Q)\left(\frac{dI_1}{dt} + (\theta + \delta + d)I_1\right) +$$
$$+Q\left(\frac{dI_1}{dt} - \frac{I_1\beta_1(N I_1 I_2 A)}{N} - \frac{I_2\beta_2(N I_1 I_2 A)}{N} + (\theta + \delta + d)I_1\right) = 0 \quad (6)$$

$$(1-Q)\left(\frac{dI_2}{dt} + (\delta + d)I_2\right) + Q\left(\frac{dI_2}{dt} - \theta I_1 + (\delta + d)I_2\right) = 0 \quad (7)$$

$$(1-Q)\left(\frac{dA}{dt} + (\alpha + d)A\right) + Q\left(\frac{dA}{dt} - \delta I_1 - \delta I_2 + (\alpha + d)A\right) = 0 \quad (8)$$

The solution of equations (5)-(8) was expressed in power series:

$$N = N_0 + QN_1 + Q^2N_2 + Q^3N_3 + \ldots \quad (9)$$
$$I_1 = I_{10} + QI_{11} + Q^2I_{12} + Q^3I_{13} + \ldots \quad (10)$$
$$I_2 = I_{20} + QI_{21} + Q^2I_{22} + Q^3I_{23} + \ldots \quad (11)$$
$$A = A_0 + QA_1 + Q^2A_2 + Q^3A_3 + \ldots \quad (12)$$

Equations (9)-(12) into equation (5)-(8) and arranging the coefficients of the powers of Q produced the following systems of differential equations:

$$Q^0: \frac{dN_0}{dt} - Q_0 + dN_0$$
$$Q^1: \frac{dN_1}{dt} + \alpha A_0 + dN_1 \quad (13)$$
$$Q^2: \frac{dN_2}{dt} + \alpha A_1 + dN_2$$

$$Q^0: \frac{dI_{10}}{dt} + (\theta + \delta + d)I_{10}$$
$$Q^1: \frac{dI_{11}}{dt} + (\theta + \delta + d)I_{11} + \beta I^2_{10} - \beta I_{10}N_0 + \beta I_{10}I_{20} +$$
$$+ \beta I_{10}A_0 + \beta I_{20} + \beta I_{20}A_0 \quad (14)$$
$$Q^2: \frac{dI_{12}}{dt} + (\theta + \delta + d)I_{12} + 2\,\beta I_{10}I_{11} - \beta I_{10}N_1 + \beta I_{10}I_{21} +$$
$$+ \beta I_{10}A_1 - \beta I_{11}N_0 + \beta I_{11}I_{20} + \beta I_{11}A_0 - \beta I_{21}N_0 - \beta I_{20}N_1 +$$
$$+ 2\beta I_{20}I_{21} + \beta I_{20}A_1 + \beta I_{21}A_0$$

$$Q^0: \frac{dI_{20}}{dt} + (\delta + d)\,I_{20}$$
$$Q^1: \frac{dI_{21}}{dt} - \theta\,I_{10} + (\delta + d)\,I_{21} \quad (15)$$
$$Q^2: \frac{dI_{22}}{dt} - \theta\,I_{11} + (\delta + d)\,I_{22}$$

$$Q^0: \frac{dA_0}{dt} + (\alpha + d)\,A_0$$
$$Q^1: \frac{dA_1}{dt} + (\alpha + d)\,A_1 - \delta\,I_{10} - \delta\,I_{20} \quad (16)$$
$$Q^2: \frac{dA_2}{dt} + (\alpha + d)\,A_2 - \delta\,I_{11} - \delta\,I_{21}$$

Table 1. Parameters values of HIV/AIDS model

Parameters	Description	Value	Source
Q_0	The rate of constant immigration of susceptible cases	200	[11]
d	The natural morality rate unrelated to HIV/AIDS	0.02	[11]
α	Deaths caused by AIDS	1.00	[11]
$\beta_{1,2}$	Contact rates for susceptible persons are anticipated to vary according on the type of relationship	1.344, 0.15	[11]
θ	The pace, at which patients become aware of being infected after a screening process	0.015	[11]
δ	Both types of infectious agents cause AIDS at different rates	0.1	[11]

The solution of analytical expression of (13)-(16) with initial condition was given by:

$$\frac{dN}{dt} = -9988.00260e^{(-0.02t)} + 10000.0 + 0.0026e^{(-1.02t)} \quad (17)$$

$$\frac{dI_1}{dt} = 0.004999999571e^{(-0.135t)} + 0.000000000046547e^{(-1.43t)} - 0.000000000036265e^{(-1.415t)} \quad (18)$$

$$\frac{dI_2}{dt} = 0.008450000000e^{(-0.12t)} - 0.05000000000e^{(-1.135t)} \quad (19)$$

$$\frac{dA}{dt} = 0.00165169 + 0.0005649e^{(-0.135t)} + 0.000383e^{(-0.12t)} \quad (20)$$

Numerical calculation and discussion

To examine the dynamical behavior of the system of equation formulation, systematic equations (1)–(4) were numerically calculated using fourth-order Runge-Kutta approach, with the following settings [11]: $Q_0 = 200$, $d = 0.02$, $\alpha = 1$, $\beta_1 = 1.344$, $\beta_2 = 0.15$, $\theta = 0.015$, $\delta = 0.1$, with the initial condition $N(0) \geq 0$, $I_2(0) \geq 0$, $I_2(0) \geq 0$, $A(0) \geq 0$.

Figures 1 and 2 show the numerical and analytical comparison of parameters Q_0 and d. By increasing the value of Q_0 into 200, 900, and 1,500, non-negative population may vary from lower-infected population into higher-infected population. In Figure 2, d is the unrelated to HIV illness natural morality rate, which is falling by 0.02, 0.05, and 0.09.

Figure 4 shows that the natural morality of infectives varies according to the values of d. The death rate of infected people decreases when the morality death increases, when the values of d is 0.02, 0.05 and 0.09 with respect to time. In Figure 6A and 6B when HIV-infective individuals engage in sexual activity without reporting their status, the number of infective people who are unaware of their status grows, resulting in an increase in AIDS population. „HIV-positive individuals who are aware of the symptoms avoid sexual intercourse". As the majority of people living with AIDS diminishes, the number of people who have no knowledge that they are infected falls. In the absence of unconscious infective screening, these figures also show.

The fluctuation in HIV-infected community awareness and ignorance is shown in Figures 3 and 7. The screening

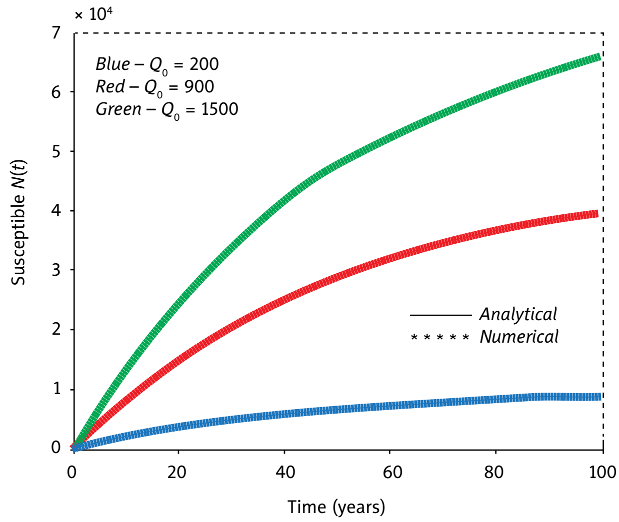

Figure 1. Variation of non-negative population for different values

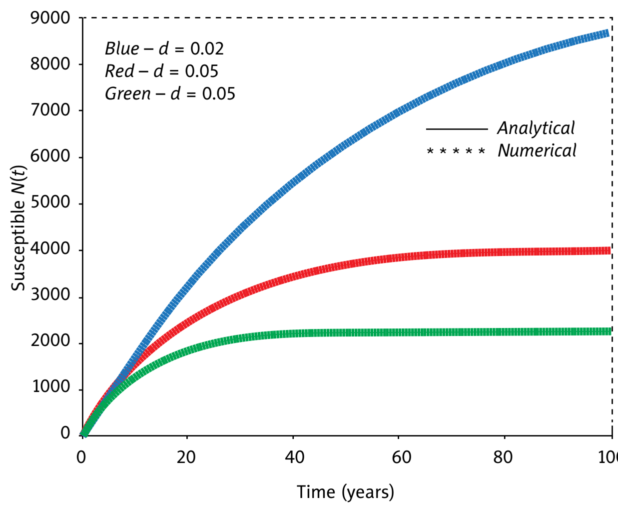

Figure 2. Natural death rate unrelated to HIV/ AIDS illness variation for different values

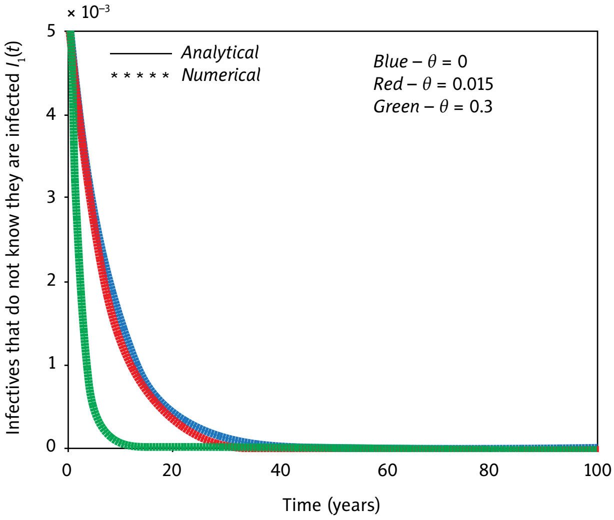

Figure 3. Variation of HIV-infected people who unaware of it (I_1) for different values of θ

rates are represented in various of formats. As the rate of HIV detection rises, previously unaware HIV infectives become aware of their infection, resulting in fewer unaware and more aware HIV infectives in (Figure 3). Figures 5, 8, and 10 illustrate that as the rate of transmission from infectious categories improves, the ignorant and aware infectious populations decrease, culminating in a rise in the AIDS demographic. The impact of disease-induced mortality rate is demonstrated in Figure 12, which illustrates that as the rates go up, the number of AIDS patients falls. Natural morality of the aware population varies for different values of d and is shown in Figures 9 and 11. The natural morality rate decreases when the value of d increase and again, there is slight difference when the value of d vary in AIDS-positive patients.

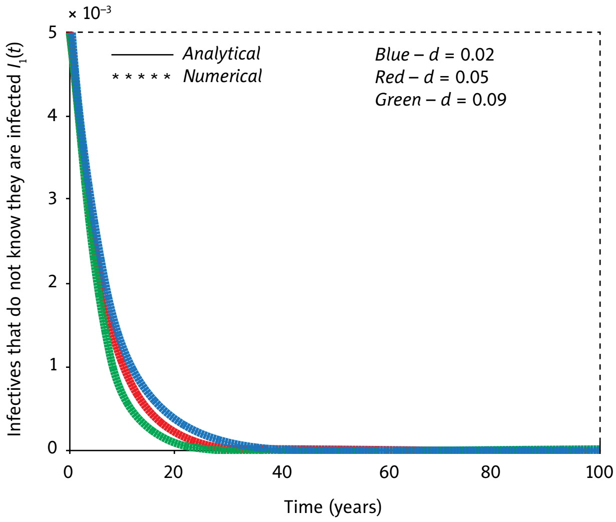

Figure 4. Natural morality varies depending on the value d

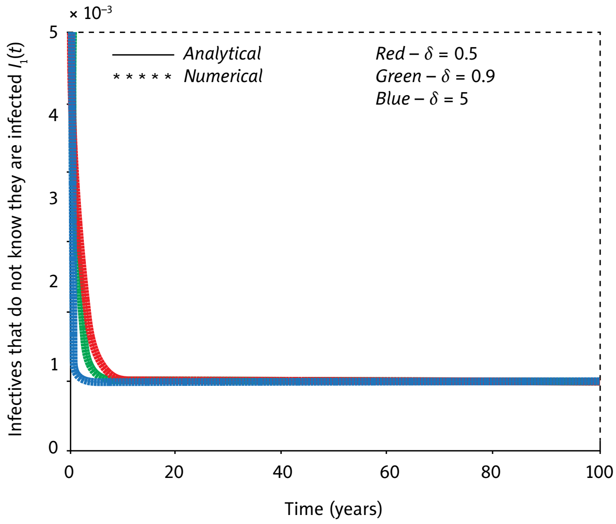

Figure 5. Variation of infective population that is ignorant for different values of δ

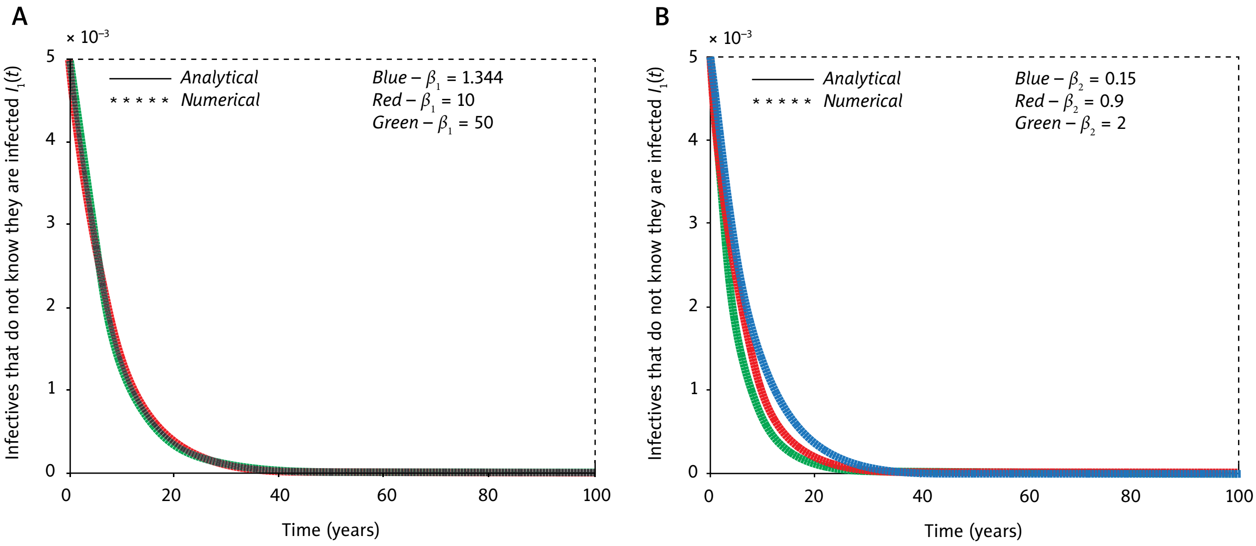

Figure 6. Variation in population contact rates for different β_1 and β_2

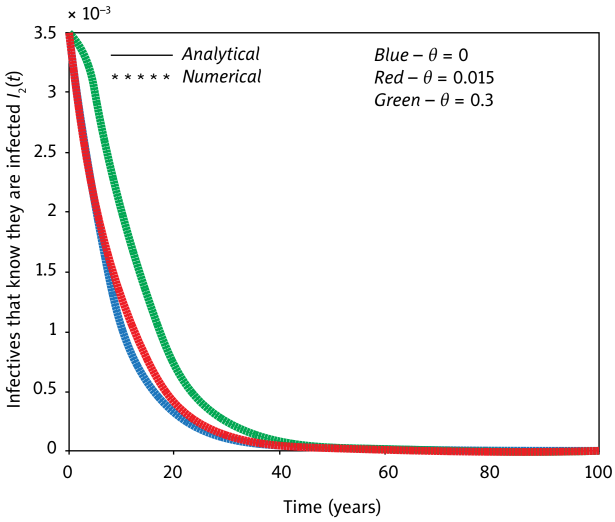

Figure 7. Variation among HIV-positive patients who are aware of their infection I_2 for different values of δ

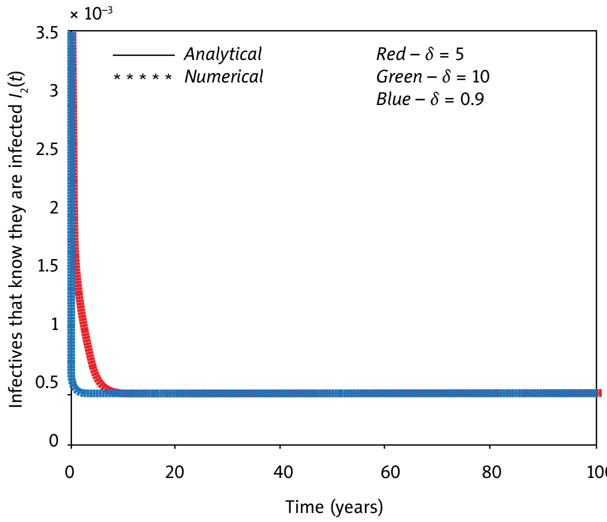

Figure 8. Variation among infective patients who are aware of their infection I_2 for different values of δ

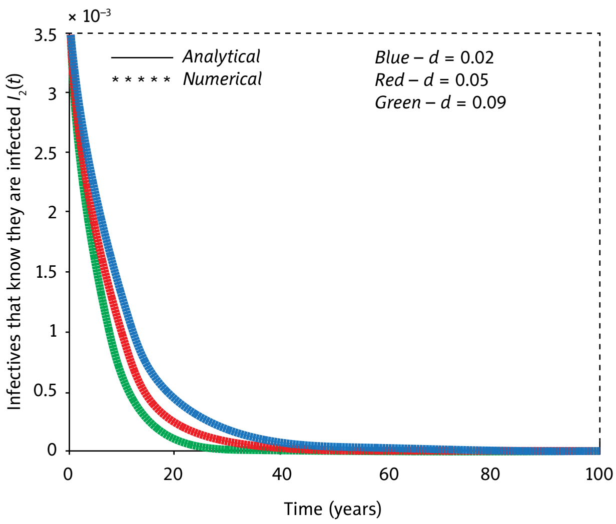

Figure 9. Natural morality of the aware population varies for different values of d

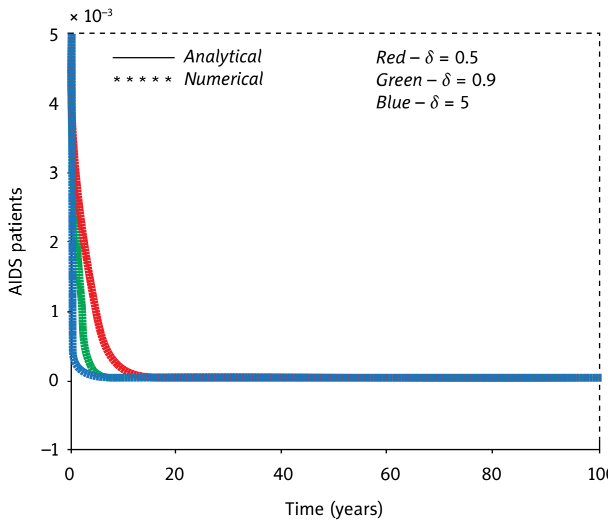

Figure 10. Variation of AIDS patients for different values of δ

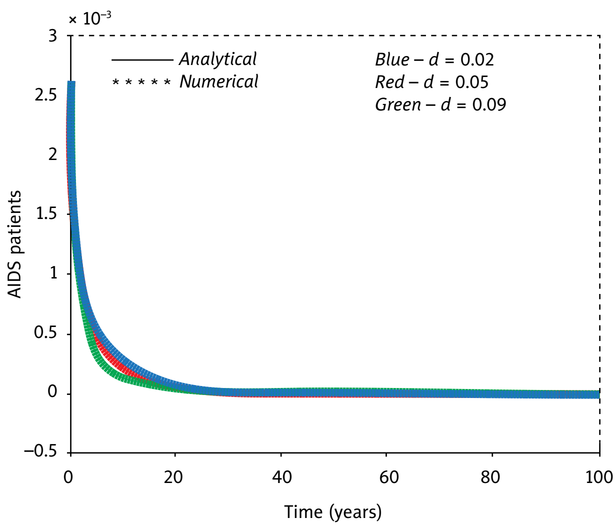

Figure 11. Variation of natural morality of AIDS patients d

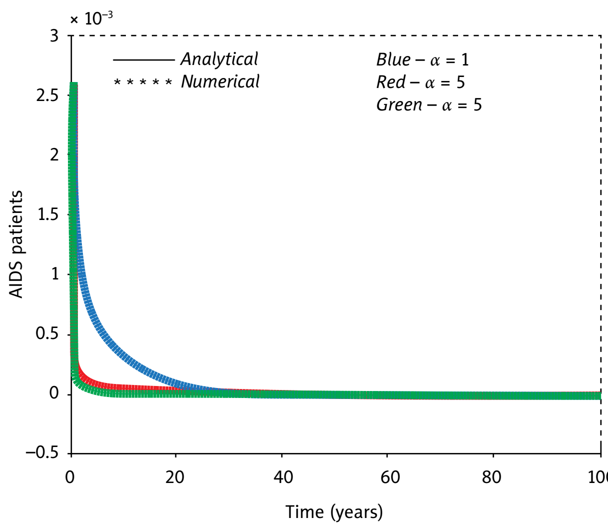

Figure 12. Variation of AIDS patients for different values of α

Conclusions

A non-linear mathematics approach is used to evaluate the impact of screening unsuspecting HIV infectives on HIV infection transmission in society with constant emigration of vulnerable people. As a result of immigration, the disease becomes more endemic; nevertheless, the disease's endemicity is controlled when infectives are conscious of their disease following testing and avoid sexual intercourse. To get analytical formula for the concentrations, a modified version of homotopy perturbation approach was applied, with four groups of individuals: Humans who are not negative – $N(t)$, those who are HIV-positive yet unaware that they are infected – $I_1(t)$, positive and aware of HIV infection – $I_2(t)$, and AIDS patients (the most advanced stage of HIV infection) – $A(t)$. MATLAB ode45 function generates numerical results that are consistent with the analytical solution. A numerical assessment of the model was also conducted using Indian HIV data to evaluate how certain key parameters affected disease transmission. The precise analytical results obtained can be utilized in sensitivity analysis of the characteristics of regulating system in order to better understand the disease's transmission mechanism and offer relevant preventative approaches.

Finally, based on the findings, it is feasible to conclude that educating the public about the dangers of non-safe sex and significance of adopting preventative steps to avoid infection is the most successful technique for lowering the incidence rate and prevalence level. Disease spread can be reduced even with low screening rates if the public has a positive attitude toward preventative practices. As a result, in order to raise awareness about the disease and prevent its' spread, education programs must reach people from all walks of life, particularly the lower classes and other high-risk groups.

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Ethical clearance

Ethical approval was obtained from the Academy of Maritime Education and Training University (AMET), Chennai, India.

Conflict of interest

The authors declare no conflict of interest.

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Appendix A.

Homotopy perturbation method's (HPM) fundamental principles

Consider the following non-linear functional equation to highlight the essential elements of this technique:

$S(v) - M(t) = 0, t \epsilon \Omega$ (A1)

$R\left(v, \frac{\partial v}{\partial n}\right) = 0, t \in \Gamma$ (A2)

S is the functional operator, *R* denotes the limit operator, *M(t)* is the known analytic capability, and *Ω* denotes the domain limit *Γ*.

L and *N* are the two halves of the operator *S*, with *L* representing linear and *N* denoting non-linear.

$L(v) + N(v) - M(t) = 0$ (A3)

We created an equation using the homotopy procedure:

$H(v, Q) = (1 - Q)[L(v) - L(v_0)] - Q (S(v) - M(t)] = 0, Q \epsilon [0.1], t \epsilon \Omega$ (A4)

or

$H(v, Q) = L(v) - L(v_0) + QL(v_0) + Q[N(v) - M(t)] = 0$ (A5)

Where $Q \epsilon 0.1$ is an parameter; for the solution of equation, v_0 is an initial approximation (A2), which meets the border requirements. Clearly, from eqns. (A4) and (A5), it may be presumed as follow:

$H(v, 0) = L(v) - L(v_0) = 0$ (A6)

$H(v, 1) = S(v) - M(t) = 0$ (A7)

Altering Q's value from zero to unity corresponds to changing:

$v(t, Q)$ value from $v_0(t)$ to $v(t)$. This is known as homotopy in topology. The embedding parameter can be used as a tiny parameter, and we can assume that the solution of eqns. (A4) and (A5) is a power series of Q:

$V = v_0 + Qv_1 + Q^2v_2 + \dots$ (A8)

Setting $Q = 1$, approximation to the solution of eqn. (A8):

$v = \lim_{P \to 1} V = v_0 + v_1 + v_2 + \dots$ (A9)

HPM is a mixture of perturbation and homotopy methods that has overcome the constraints of classic perturbation approaches. For additional situations, the series eqn. (A9) is convergent.